## PRE BOARD-2, (2024-25)

APPLIED MATHEMATICS (241)

Marking Scheme

### Section A

1.

**(c)** p = 2, q = 4

**Explanation:** $A_{5\times p} \times B_{2\times q} = C_{5\times q}$

$\Rightarrow$ p = 2

Order of matrix.

$(AB)_{5\times 4} = C_{5\times q}$

$\Rightarrow$ q = 4

2.

**(c)** estimating a statistic

**Explanation:** estimating a statistic

3. **(a)** ₹62500

**Explanation:** ₹62500

4.

**(d)** every point on the line segment joining the points (0, 2) and (3, 0)

**Explanation:** every point on the line segment joining the points (0, 2) and (3, 0)

5.

**(b)** Option (d)

**Explanation:** to each element of a row (or a column) is added equi-multiples of the corresponding elements of another row or column.

6. **(a)** $\frac{2}{3}$

**Explanation:** Probability of getting a doublet $= p = \frac{6}{36} = \frac{1}{6}$

A pair of dice is thrown four times events are independent, therefore, it is a problem of binomial distribution.

Here, $p = \frac{1}{6}$, n = 4.

So, mean = np $= 4 \times \frac{1}{6} = \frac{2}{3}$.

7. **(a)** 7, 14

**Explanation:** Here, $p = \frac{1}{2}$ and $q = \frac{1}{2}$

Binomial distribution is given by

P(X = r) $= {}^n C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r}$

P(X = 4), P(X = 5), P(X = 6) are in A.P.

$\therefore {}^nC_4 + {}^nC_6 = 2\,{}^nC_5$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{2(4!)} + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{2(6!)} = \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}$

By simplifying, we get

$\frac{1}{2} + \frac{(n-4)(n-5)}{2(30)} = \frac{n-4}{5}$

Taking LCM as 60, we get

$30 + n^2 - 9q + 20 = 12n - 48$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow (n - 7)(n - 14) = 0$

$\Rightarrow$ n = 7, 14

8.

**(d)** not defined

**Explanation:** As the term $\log\left(\frac{d^2y}{dx^2}\right)$ is not a polynomial in $\frac{d^2y}{dx^2}$. So, the degree of the given differential equation is not defined.

9.

**(c)** 200 minutes

**Explanation:** Suppose pipe A alone takes x hours to fill the tank.

Then pipes B and C will take $\frac{x}{3}$ and $\frac{x}{6}$ hours respectively to fill the tank.

$\therefore \frac{1}{x}+\frac{3}{x}+\frac{6}{x}=\frac{1}{20}$

$\Rightarrow \frac{10}{x}=\frac{1}{20}$

$\Rightarrow$ x = 200 hours

$\therefore$ pipe A alone will take 200 hours to fill the tank.

10.

**(d)** none of these

**Explanation:** We have,

A = $\begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$

$A^{-1}$ exists if |A| $\neq$ 0

Now |A| = 2(6 - 5) - $\lambda$(-5) - 3(-2) = 8 + 5$\lambda \neq$ 0

$\Rightarrow$ 5$\lambda \neq$ -8

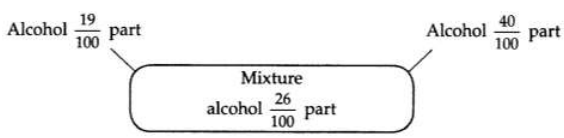
$\Rightarrow \lambda \neq \frac{-8}{5}$

So, $A^{-1}$ exists if and only if $\neq \frac{-8}{5}$

11.

**(b)** $\frac{2}{3}$ part

**Explanation:**




So, ratio $\frac{\frac{40}{100}-\frac{26}{100}}{\frac{26}{100}-\frac{19}{100}}=\frac{14}{7}=\frac{2}{1}$.

$\therefore$ The quantity of whisky replaced by 19% alcohol $=\frac{2}{2+1}$ i.e. $\frac{2}{3}$ part

12.

**(d)** x $\in(-\infty$, -b) $\cup$ (b, $\infty$)

**Explanation:** x $\in(-\infty$, -b) $\cup$ (b, $\infty$)

13.

**(c)** 8 km

**Explanation:** The speed of boat in still water = 15 km/hr

Speed of water current = 5 km/hr

$\therefore$ Speed in down stream = 15 + 5 = 20 km/hr

Time given = 24 min = $\frac{24}{60}$ hr = $\frac{2}{5}$ hr

$\therefore$ Distance travelled = speed $\times$ times

= 20 $\times \frac{2}{5}$ = 8 km

14.

**(c)** at an infinite number of points

**Explanation:**

Given the objective function is Z = 4x + 3y

constraints are:

3x + 4y $\leq$ 24

8x + 6y $\leq$ 48

x $\leq$ 5

y $\leq$ 6

x $\geq$ 0

y $\geq$ 0

If we consider these inequalities as equalities for some time,

We will have
3x + 4y = 24
8x + 6y = 48
x = 5
y = 6
x = 0
y = 0
If we plot all these lines on a graph we will have optimal area formed by the vertices, OABCD.

Now, to find where the function Z has maximized, let us substitute all these points in the objective function Z.

| | |
|---|---|
| Z at O(0, 0) | Z = 4(0) + 3(0) = 0 + 0 |
| Z at A(0, 6) | Z = 4(0) + 3(6) = 0 + 18 = 18 |
| Z at B$\left(\frac{24}{7}, \frac{24}{7}\right)$ | Z = 4$\left(\frac{24}{7}\right)$ + 3$\left(\frac{24}{7}\right)$ = $\frac{96+72}{7} = \frac{168}{7}$ = 24 |
| Z at C$\left(5, \frac{4}{3}\right)$ | Z = 4(5) + 3$\left(\frac{4}{3}\right)$ = 20 + 4 = 24 |
| Z at D(5, 0) | Z = 4(5) + 3(0) = 20 + 0 = 20 |

Here, we can clearly see that, the function Z is maximized at two points B & C giving the value 24.
There will be infinite/multiple optimal solutions for a LPP if it has more than one set of optimal solutions that can maximize/ minimize a problem.
This will clear the fact that, the function Z will maximize at infinite number of points.

15. **(a)** 5
**Explanation:** (1, 1), (1, -1), (-1 , -1), (2, -1) and (-2, -1) satisfy the inequality 2x - 3y > - 5.

16.
**(c)** hypothesis
**Explanation:** hypothesis

17.
**(d)** 27
**Explanation:** Given p = 20 - 2x - $x^2$ and $x_0$ = 3
So, $p_0$ = 20 - 2 × 3 - 32 ⇒ $p_0$ = 5
$CS = \int_0^3 \left(20 - 2x - x^2\right) dx - 3 \times 5$
$= \left[20x - x^2 - \frac{x^3}{3}\right]_0^3 - 15$ = (60 - 9 - 9) - 15 = 27

18. **(a)** Secular trend
**Explanation:** In seasonal variation, tendency movements are due to the nature which repeat themselves periodically in every season.

19. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:** Given AB is invertible ⇒ |AB| $\neq$ 0

$\Rightarrow$ |A|·|B| $\neq$ 0 $\Rightarrow$ |A| $\neq$ 0

$\Rightarrow$ A is invertible $\Rightarrow$ Reason is true.

Now, AB = AC (given)

Pre-multiplying by $A^{-1}$ both the sides ($\because$ |A| $\neq$ 0 $\Rightarrow$ $A^{-1}$ exists)

$A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1} A) B = (A^{-1} A)C \Rightarrow IB = IC$

$\Rightarrow$ B = C

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** f(x) = log IxI + $bx^2$ + ax, x $\neq$ 0

$\Rightarrow$ f'(x) = $\frac{1}{x}$ + 2bx + a, x $\neq$ 0

Given x = -1 and x = 2 are extreme values of f(x).

So, f'(-1) = 0 and f'(2) = 0

$\Rightarrow$ -1 - 2b + a = 0 and $\frac{1}{2}$ + 4b + a = 0

Solving these equations, we get a = $\frac{1}{2}$, b = $-\frac{1}{4}$

$\therefore$ Reason is true.

Now, f'(x) $\frac{1}{x} - \frac{1}{2}x + \frac{1}{2}$ = $\frac{2-x^2+x}{2x}$

$\Rightarrow$ f'(x) = 0 $\Rightarrow$ x = -1 and x = 2

f''(x) = $-\frac{1}{x^2} - \frac{1}{2}$ $\Rightarrow$ f''(-1) = $\frac{1}{1} - \frac{1}{2} = -\frac{3}{2}$ < 0

$\Rightarrow$ x = -1 is a point of local maxima.

Also f''(2) = $-\frac{1}{4} - \frac{1}{2} = -\frac{3}{4}$ < 0

$\Rightarrow$ x = 2 is a point of local maxima.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

**Section B**

21. Here we have to find how much money should be invested now that would provide for an unlimited number of payments of ₹2,500 each year, the first due now. So, it is a perpetuity of ₹2,500 payable at the beginning each year, if money is worth 5% compounded annually. Thus, we have

R = 2,500 and i = $\frac{5}{100}$ = 0.05

Let P be the present value of this annuity. Then,

P = R + $\frac{R}{i}$ $\Rightarrow$ P = ₹(2,500 + $\frac{2,500}{0.05}$) = ₹52,500

Hence, required sum of money is ₹52,500.

OR

P = ₹ 150000, i = $\frac{12}{1200}$= = 0.01, n = 10 $\times$ 12 = 120

i. EMI = $\frac{P}{a_{\bar{n}/i}} = \frac{150000}{a_{120/0.01}} = \frac{150000}{69.6891}$ = ₹ 2152.43.

ii. Total interest paid = n $\times$ EMI - P

= 120 $\times$ 2152.42 - 150000

= ₹108290.40

22. Construction of 3-yearly moving average

| Year | Imported cotton consumption in India (in '000 bales) | 3-yearly moving totals | 3-yearly moving averages |
|---|---|---|---|
| 2010 | 129 | - | - |
| 2011 | 131 | 366 | 122.00 |
| 2012 | 106 | 328 | 109.33 |
| 2013 | 91 | 292 | 97.33 |
| 2014 | 95 | 270 | 90.00 |
| 2015 | 84 | 272 | 90.66 |
| | | | |

| | | | |
|---|---|---|---|
| 2016 | 93 | - | - |

23. First note that the given function is discontinuous at x = 1.

$\therefore \int_{-1}^{2} f(x)dx = \int_{-1}^{1} f(x)dx + \int_{1}^{2} f(x)dx$

= $\int_{-1}^{1}(2x+1)dx + \int_{1}^{2}(x-5)dx$

= $\left[x^2+x\right]_{-1}^{1} + \left[\frac{x^2}{2} - 5x\right]_1^2$

= (1 + 1) - (1 - 1) + (2 - 10) $-\left(\frac{1}{2}-5\right)$

= 2 - 0 - 8 $+\frac{9}{2} = -\frac{3}{2}$

24. We are given that, $A^2 = I$

Now, $(A - I)^3 + (A - I)^3 - 7A$

$= (A^3 - 3A^3I + 3AI^3 - I^3) + (A^3 + 3A^3I + 3AI^3 + I^3) - 7A$

$= A^3 - 3A^3I + 3AI^3 - I^3 + A^3 + 3A^3I + 3AI^3 + I^3 - 7A$

$= 2A^3 + 6AI - 7A$

$= 2A^2A + 6A - 7A$ ... [$\because$ AI = A]

= 2IA - A

= 2A - A ...[$\because$ AI = A]

= A

OR

Given,

$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -2x+0+7x & 28x+0-28x & 14x+0-14x \\ 0+0+0 & 0+1+0 & 0+0+0 \\ -x+0+x & 14x-2-4x & 7x+0-2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since, corresponding entries of equal matrices are equal, so

5x = 1 and 10x - 2 = 0

$\Rightarrow$ x = $\frac{1}{5}$ and x = $\frac{1}{5}$

Hence, x = $\frac{1}{5}$.

25. Let us first find the gcd of 27 and 392 by using the Euclidean algorithm.

392 = 27 $\times$ 14 + 14 [Dividing 392 by 27]

27 = 14 $\times$ 1 + 13

14 = 13 $\times$ 1 + 1

Using reverse substitution, we obtain

1 = 14 - 13 $\times$ 1

$\Rightarrow$ 1 = 14 - (27 - 14 $\times$ 1) $\times$ 1

$\Rightarrow$ 1 = 14 $\times$ 2 - 27 $\times$ 1

$\Rightarrow$ 1 = (392 - 27 $\times$ 14) $\times$ 2 - 27 $\times$ 1

$\Rightarrow$ 1 = 392 $\times$ 2 - 27 $\times$ 29

$\Rightarrow$ 1 = 27 $\times$ (-29) + 392 $\times$ 2

Thus, -29 is the multiplicative inverse of 27 under modulo 392. But, -29 = 363 (mod 392) and the inverse must be in the set (0, 1, 2, ..., 391). Hence, 363 is required multiplicative inverse.

**Section C**

26. We are given that

C = 30,000; n = 4; S = 4000

Annual depreciation = $\frac{C-S}{n}$

$= \frac{30000-4000}{4}$

= 6500

Depreciation schedule

| **Year** | **Annual depreciation** | **Accumulated depreciation** | **Book Value** |
|---|---|---|---|
| 0 | 0 | 0 | 30,000 |
| 1 | 6500 | 6500 | 23,500 |
| 2 | 6500 | 13000 | 17,000 |
| 3 | 6500 | 19,500 | 10,500 |
| 4 | 6500 | 26,000 | 4000 |

27. The given differential equation is

x log x$\frac{dy}{dx}$ + y = $\frac{2}{x}$ log x

⇒ $\frac{dy}{dx}+\frac{1}{x\log x}y=\frac{2}{x^2}$ ...(i)

This is a linear differential equation of the form

$\frac{dy}{dx}$ + Py = Q, where P = $\frac{1}{x\log x}$ and Q = $\frac{2}{x^2}$

∴ I.F. = $e^{\int Pdx}=e^{\int\frac{1}{x\log x}dx}=e^{\int\frac{1}{t}dt}$ , where t = log x

⇒ I.F. = $e^{\log t}$ = t = log x

Multiplying both sides of (i) by I.F. = log x, we get

log x$\frac{dy}{dx}+\frac{1}{x}$y = $\frac{2}{x^2}$ log x

Integrating both sides with respect to x, we get

y log x = $\int\frac{2}{x^2}$ log x dx + C [Using: y(I.F.) = $\int$ Q (I.F.) dx + c]

⇒ y log x = $2\int\underset{I}{\log x}\,\underset{II}{x^{-2}}dx+C$

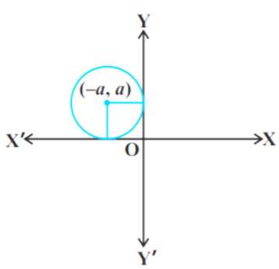
⇒ y log x = $2\left\{\log x\left(\frac{x^{-1}}{-1}\right)-\int\frac{1}{x}\left(\frac{x^{-1}}{-1}\right)dx\right\}$ + C

⇒ y log x = 2$\left\{-\frac{\log x}{x}+\int x^{-2}dx\right\}$ + C

⇒ y log x = 2$\left\{-\frac{\log x}{x}-\frac{1}{x}\right\}$ + C

⇒ y log x = $-\frac{2}{x}$ (1 + log x) + C, which gives the required solution.

OR




Eq. of circle is

$(x + a)^2 + (y - a)^2 = a^2$ ...(1)

⇒ $x^2 + y^2 + 2ax - 2ay + a^2 = 0$

Differentiating both sides w.r.t x, we get,

2x + 2yy' + 2a - 2ay' = 0

⇒ x + yy'= a(y' - 1)

⇒ $\frac{x+yy'}{y'-1}$ = a

Put the value of a in eq (1), we get,

$(x+\frac{x+yy'}{y'-1})^2+(y-\frac{x+yy'}{y'-1})^2=(\frac{x+yy'}{y'-1})^2$

⇒ $\left(\frac{x(y'-1)+x+yy'}{y'-1}\right)^2+\left(\frac{y(y'-1)-x-yy'}{y'-1}\right)^2=\left(\frac{x+yy'}{y'-1}\right)^2$

⇒ $\left(\frac{xy'-x+x+yy'}{y'-1}\right)^2+\left(\frac{-x-y}{y'-1}\right)^2=\left(\frac{x+yy'}{y'-1}\right)^2$

⇒ $\left(\frac{(x+y)y'}{y'-1}\right)^2+\left(\frac{-(x+y)}{y'-1}\right)^2=\left(\frac{x+yy'}{y'-1}\right)^2$

$\Rightarrow y^2(x + y)^2 + (x + y)^2 = (x + yy')^2$

$\Rightarrow (x + y)^2(y'^2 + 1) = (x + yy')^2$

28. i. Let P, C, and R be the profit function, the cost function and the revenue function respectively. It is given that

MC = 16x - 1591

$\Rightarrow \frac{dC}{dx} = 16x - 1591$

Integrating both sides with respect to x, we obtain

$C = \int(16x - 1591)dx$

$\Rightarrow C = 8x^2 - 1591x + K$ ...(i)

where K is an arbitrary constant. It is given that the fixed cost is ₹ 800 i.e. C = 1800 when x = 0. Substituting these values in (i), we obtain K = 1800. Therefore, the cost function is given by

$C = 8x^2 - 1591x + 1800$

ii. The selling price is fixed at ₹ 9 per unit. So, the revenue function R is given by R = 9x

iii. the profit function P is given by

P = R - C

$\Rightarrow P = 9x - (8x^2 - 1591x + 1800)$

$\Rightarrow P = -8x^2 + 1600x - 1800$

iv. We have,

$P = -8x^2 + 1600x - 1800$

$\Rightarrow \frac{dP}{dx} = -16x + 1600$ and $\frac{d^2P}{dx^2} = -16$

For maximum profit, we must have

$\frac{dP}{dx} = 0 \Rightarrow -16x + 1600 = 0 \Rightarrow x = 100$

Clearly, $\frac{d^2P}{dx^2} = -16 < 0$ for all x.

Thus, P is maximum when x = 100. The corresponding profit is given by

$P = -8 \times 100^2 + 1600 \times 100 - 1800 = 78,200$

Hence, maximum profit = ₹ 78,200

29. Computation of three-yearly moving averages

| **Years** | **Profit (₹)** | **3-yearly moving Total (₹)** | **3-yearly moving average (₹)** |
|---|---|---|---|
| 2016 | 15,420 | - | - |
| 2017 | 15,470 | 46,410 | 15,470 |
| 2018 | 15,520 | 52,010 | 17,336.667 |
| 2019 | 21,020 | 63,040 | 21,013.333 |
| 2020 | 26,500 | 79,470 | 26,490 |
| 2021 | 31,950 | 94,050 | 31,350 |
| 2022 | 35,600 | 94,050 | 31,350 |
| 2023 | 34,900 | - | - |

The last column gives the trend of profits.

30. We have,

$\mu$ = Population mean = 100, n = Sample size = 10

We define

Null Hypothesis $H_0$: The data are consistent with the assumption of a mean I.Q. of 100 in the population.

Alternate hypothesis $H_1$: The mean I.Q. of population $\neq$ 100

Let the sample statistic t be given by

$t = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}$, where $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{X}\right)^2$

Let us now compute $\bar{X}$ and $S^2$.

Computation of $\bar{X}$ and S

| $x_i$ | $d_1 = x_i - 90$ | $d_i^2$ |
|---|---|---|
| 70 | -20 | 400 |
| 120 | 30 | 900 |
| 110 | 20 | 400 |
| 101 | 11 | 121 |
| 88 | -2 | 4 |
| 83 | -7 | 49 |
| 95 | 5 | 25 |
| 98 | 8 | 64 |
| 107 | 17 | 289 |
| 100 | 10 | 100 |
| | $\sum d_i = 72$ | $\sum d_i^2 = 2352$ |

Here, $d_i = x_i - 90$

$\therefore \bar{X} = 90 + \frac{1}{10}\sum d_i = 90 + \frac{72}{10} = 972$ [Using : $\bar{X} = A + \frac{1}{n}\sum d_i$]

$S^2 = \frac{1}{n-1}\left\{\sum d_i^2 - \frac{1}{n}\left(\sum d_i\right)^2\right\} = \frac{1}{9}\left\{\mathbf{2352} - \frac{(72)^2}{10}\right\} = \frac{1833.6}{9} = 203.73$

$\therefore t = \frac{\bar{X}-\mu}{S/\sqrt{n}} \Rightarrow t = \frac{97.2-100}{\sqrt{\frac{203.73}{10}}} = \frac{-2.8}{\sqrt{20.37}} = \frac{-2.8}{4514} = -0.62$

$\Rightarrow |t| = 0.62$

The sample statistict follows student's t -distribution with v = (10 - 1) = 9 degrees of freedom. It is given that $t_9(0.05) = 2.262$

$\therefore$ Calculated $|t|$ < tabulated $t_9(0.05)$

So, the null hypothesis may be accepted at 5% level of significance.

Hence, the assumption of a population mean I.Q. of 100 is valid.

The 95% confidence limits within which the mean I.Q. values of samples of 10 boys will lie are

$\bar{X} - \frac{S}{\sqrt{n}} t_9(0.05)$ and $\bar{X} + \frac{S}{\sqrt{n}} t_9(0.05)$

or $97.2 - \sqrt{\frac{203.73}{10}} \times 2.262$ and $97.2 + \sqrt{\frac{203.73}{10}} \times 2.262$

or, $97.2 - 4514 \times 2.262$ and $97.2 + 4.514 \times 2.262$

or, 97.2 - 10.21 and 97.2 + 10.21

or, 86.99 and 107.41

Hence, the required 95% confidence interval is [86.99, 107.41]

31. Let X denotes the number of kings in a draw of two cards. X is a random variable which can assume the values 0,1 or 2.

Then P(X = 0) = P(no card is king)

$= \frac{^{48}C_2}{^{52}C_2}$

$= \frac{48\times47}{52\times51}$

$= \frac{188}{221}$

P(x = 1) = P(exactly one card is king)

$= \frac{^{4}C_1\times{}^{48}C_1}{^{52}C_2}$

$= \frac{4\times48\times2}{52\times51}$

$= \frac{32}{221}$

P(X = 2) = P(both cards are king)

$= \frac{^{4}C_2}{^{52}C_2}$

$= \frac{4\times3}{52\times51}$

$= \frac{1}{221}$

E(X) = $\sum_{i=1}^{n} x_i p_i$

= $0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$

= $\frac{34}{221}$

Var(X) = $\left(\sum_{i=1}^{n} x_i^2 p_i\right) - \left(\sum_{i=1}^{n} x_i p_i\right)^2$

= $\left(0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221}\right) - \left(\frac{34}{221}\right)^2$

= $\frac{36}{221} - \frac{1156}{48841}$

= $\frac{6800}{48841}$

= 0.1392

$\sigma = \sqrt{\text{Var(X)}}$

= $\sqrt{0.1392}$

= 0.3730

OR

Let n and p be the parameters of the distribution. Then,

Mean = np and Variance = npq

It is given that,

Mean + Variance = 15 and, $(\text{Mean})^2 + (\text{Variance})^2 = 117$

Now, np + npq = 15 and $n^2p^2(1 + q^2) = 117$

$\Rightarrow$ np(1 + q) = 15 and $n^2p^2 + n^2p^2q^2 = 117$

$\Rightarrow$ $n^2p^2(1 + q)^2 = 225$ and $n^2p^2(1 + q^2) = 117$

$\Rightarrow \frac{n^2p^2(1+q)^2}{n^2p^2(1+q^2)} = \frac{225}{117}$

$\Rightarrow \frac{(1+q)^2}{(1+q^2)} = \frac{225}{117}$

$\Rightarrow \frac{1+q^2+2q}{1+q^2} = \frac{225}{117}$

$\Rightarrow 1 + \frac{2q}{1+q^2} = \frac{225}{117}$

$\Rightarrow \frac{2q}{1+q^2} = \frac{108}{117}$

$\Rightarrow \frac{2q}{1+q^2} = \frac{12}{13}$

$\Rightarrow \frac{1+q^2}{2q} = \frac{13}{12}$

$\Rightarrow \frac{1+q^2+2q}{1+q^2-2q} = \frac{13+12}{13-12}$ [Applying componendo and dividendo]

$\Rightarrow \left(\frac{1+q}{1-q}\right)^2 = 25 \Rightarrow \frac{1+q}{1-q} = 5 \Rightarrow$ 6q = 4 $\Rightarrow q = \frac{2}{3} \Rightarrow p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$

Putting $p = \frac{1}{3}, q = \frac{2}{3}$ in np + npq = 15, we get

$\frac{n}{3} + \frac{2n}{9} = 15 \Rightarrow \frac{5n}{9} = 15 \Rightarrow$ n = 27

Thus, n = 27, $p = \frac{1}{3}$ and $q = \frac{2}{3}$

Hence, the distribution is given by,

P(X = r) $= {}^{27}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{27-r}$, r = 0, 1, 2, .... 27

**Section D**

32. Given,

Z = 3x + 5y

Constraints:

$-2x + y \leq 4$

$x + y \geq 3$,

$x - 2y \leq 2$,

$x, y \geq 0$

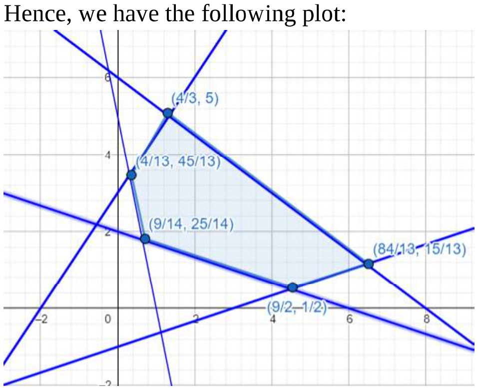
First convert the given inequations into corresponding equations and plot them:

$-2x + y \leq 4 \rightarrow -2x + y = 4$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 $\Rightarrow$ y = 4(0, 4) ...first coordinate.

Put, y = 0 ⇒ x = -2(-2, 0) ...second coordinate
Join them to get the line.
As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation.
If the given line does not pass through origin then just put (0, 0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.
Similarly, we repeat the steps for other inequations also and find the common region.
x + y $\geq$ 3 $\rightarrow$ x + y = 3 (corresponding equation)
Two coordinates required to plot the equation are obtained as:
Put, x = 0 ⇒ y = 3(0, 3) ...first coordinate.
Put, y = 0 ⇒ x = 3(3, 0) ...second coordinate
x - 2y $\leq$ 2 $\rightarrow$ x - 2y = 2 (corresponding equation)
Two coordinates required to plot the equation are obtained as:
Put, x = 0 ⇒ y= -1(0, -1) ...first coordinate.
Put, y = 0 ⇒ x = 2(2, 0) ...second coordinate
x = 0 is the y-axis and y = 0 is the x-axis.
Hence, we have the following plot:




There is no shaded region in the above figure represents that there is no region of a feasible solution.
x + y $\geq$ 3 will not be bounded by x, y $\leq$ 0. Thus, no feasible region is there.
$\therefore$ There is no possible minimum value Z.

OR

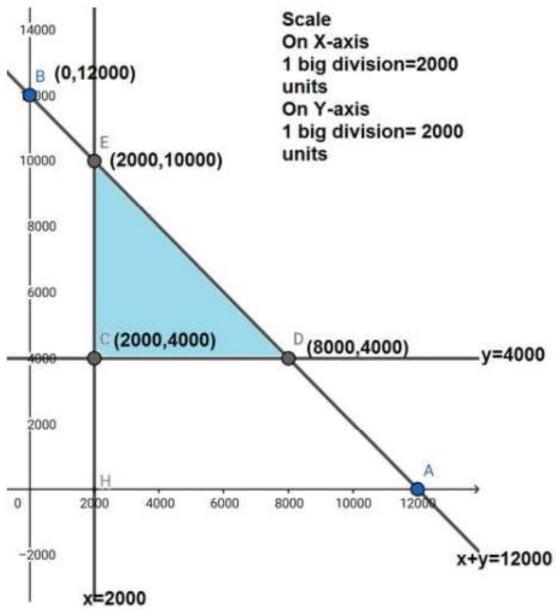
Let Anil invests ₹x and ₹y in saving certificate (SC) and National saving bond (NSB) respectively.
Since the rate of interest on SC is 8% annual and on NSB is 10% annual. So, interest on ₹ x of SC is $\frac{8x}{100}$ and ₹ y of NSB is $\frac{10x}{100}$ per annum.
Let Z be the total interest earned so,
Z = $\frac{8x}{100} + \frac{10x}{100}$
Given he wants to invest ₹12000 in total
x + y $\leq$ 12000
According to the rules he has to invest at least ₹2000 in SC and at least ₹4000 in NSB
x $\geq$ 2000
y $\geq$ 4000
Hence the mathematical formulation of LPP is to find x and y which Maximizes Z.
Max Z = $\frac{8x}{100} + \frac{10x}{100}$
Subject to constraints
x $\geq$ 2000
y $\geq$ 4000
x + y $\leq$ 12000
x, y $\geq$ 0
The region represented by x $\geq$ 2000: line x = 2000 is parallel to the y-axis and passes through (2000, 0)
The region not containing the origin represents x $\geq$ 2000
As (0, 0) doesn't satisfy the inequation x $\geq$ 2000
The region represented by y $\geq$ 4000: line y = 4000 is parallel to the x - axis and passes through (0, 4000)
The region not containing the origin represents y $\geq$ 4000

As (0, 0) doesn't satisfy the inequation $y \geq 4000$

Region represented by $x + y \leq 12000$: line $x + y = 12000$ meets axes at A(12000, 0) and B(0, 12000) respectively. The region which contains the origin represents the solution set of $x + y \leq 12000$

as (0, 0) satisfies the inequality $x + y \leq 12000$

Region $x, y \geq 0$ is represented by the first quadrant




The corner points are E(2000, 10000), C(2000, 4000), D(8000, 4000)

The values of Z at these corner points are as follows:

| Corner Points | $Z = \frac{8x}{100} + \frac{10x}{100}$ |
|---|---|
| O | 0 |
| E | 1160 |
| D | 1040 |
| C | 560 |

The maximum value of Z is ₹1160 which is attained at E(2000, 10000)

Thus the maximum earning is ₹1160 obtained when ₹2000 were invested in SC and ₹10000 in NSB.

33. Let x be the random variable denoting the number of times an odd number (the number of successes) when a die is tossed twice.

Then x takes the values 0, 1, 2

Let P(X = 0) be probability of getting no odd number (both times showing even).

$\therefore P(X = 0) = \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

Let P(X = 1) be probability of getting odd number once.

$\therefore P(X = 1) = {}^2C_1 \frac{3}{6} \times \frac{3}{6} = \frac{6}{6} \times \frac{3}{6} = \frac{1}{2}$

Let P(X = 2) be probability of getting odd number twice.

$\therefore P(X = 2) = \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

Thus the probability distribution of X is given by

X = x: x = 0, x = 1, x = 2

P(X = x) $\frac{1}{4} \frac{1}{2} \frac{1}{4}$

We know that mean $E(X) = \sum x_i p_i = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$

$\therefore E(X) = 0 + \frac{1}{2} + \frac{1}{2} = 1$

Thus mean E(X) = 1

We know that $var(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \sum x_i^2 p_i = 0 \times \frac{1}{4} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{4}$

$\therefore E(X^2) = 0 + \frac{1}{2} + 4 \times \frac{1}{4} = \frac{3}{2}$

Thus var(X) = $\frac{3}{2}$ - $[1]^2$ = $\frac{3}{2}$ - 1 = $\frac{1}{2}$

Hence mean is 1 and variance is $\frac{1}{2}$

OR

Total number of balls = 5 + 2 = 7

Two balls are drawn at random.

Let X denote the number of black balls drawn, then X can take values 0, 1, 2.

P(X = 0) = P(no black ball) = $\frac{{}^5C_2}{{}^7C_2} = \frac{5.4}{7.6} = \frac{10}{21}$

P(X = 1) = P(one black ball) = $\frac{{}^5C_1 \times {}^2C_1}{{}^7C_2} = \frac{10}{21}$

P(X = 2) = P(two black balls) $= \frac{{}^2C_2}{{}^7C_2} = \frac{1}{21}$ .

$\therefore$ Probability distribution of the number of black balls drawn is $\begin{pmatrix} 0 & 1 & 2 \\ \frac{10}{21} & \frac{10}{21} & \frac{1}{21} \end{pmatrix}$.

We construct the following table:

| $x_i$ | $p_i$ | $p_i x_i$ | $p_i x_i^2$ |
|---|---|---|---|
| 0 | $\frac{10}{21}$ | 0 | 0 |
| 1 | $\frac{10}{21}$ | $\frac{10}{21}$ | $\frac{10}{21}$ |
| 2 | $\frac{1}{21}$ | $\frac{2}{21}$ | $\frac{4}{21}$ |
| Total | | $\frac{12}{21}$ | $\frac{14}{21}$ |

Mean $= \Sigma p_i x_i = \frac{12}{21} = \frac{4}{7}$.

Variance $\Sigma p_i \dot{x}_i^2 - (\Sigma p_i x_i)^2 = \frac{14}{21} - \left(\frac{4}{7}\right)^2$

$= \frac{2}{3} - \frac{16}{49} = \frac{98-48}{147} = \frac{50}{147}$.

34. The given inequalities are

x + y $\geq$ 5 ...(i)

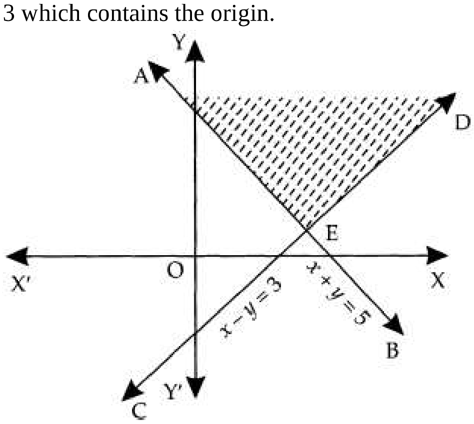
and x - y $\leq$ 3 ...(ii)

To draw the graph of x + y $\geq$ 5, we draw the straight line x + y = 5 which passes through the points (5, 0) and (0, 5). The line divides the plane into two parts.

Further as O(0, 0) does not satisfy the inequality x + y $\geq$ 5

($\because$ 0 + 0 $\geq$ 5 i.e. 0 $\geq$ 5, which is not true), therefore, the graph of (i) consists of that part of the plane divided by the line x + y = 5 which does not contain the origin.

Similarly, draw the graph of the inequality x - y $\leq$ 3. The graph of (ii) consists of that part of the plane divided by the line x - y = 3 which contains the origin.




Shade the common part of the graphs of both the given inequalities. The solution set (region) of the given inequalities consists of all points in the shaded part of the coordinate plane shown in fig. The points on the ray AE and the points on the ray ED are included in the solution.

35. Given,

Principle P = 500000

i = Rate of interest per rupee per month

= $\frac{10}{1200} = \frac{1}{120}$

n = Number of installments = 60

$EMI = P\left(i + \frac{1}{n}\right)$

$= 500000\left(\frac{1}{120} + \frac{1}{60}\right)$

$= 500000\left(\frac{1}{40}\right)$

= 12500

The EMI for a loan is ₹ 12500.

## Section E

36. i. 70xy
   ii. 90(x + y)
   iii. 90(1 - $\frac{9}{x^2}$)

   **OR**

   3

37. i. Given, P = ₹ 250000, i = $\frac{6}{12\times100}$ = 0.005 and n = 5 × 12 = 60

   EMI = $\frac{250000\times0.005\times(1.005)^{60}}{(1.005)^{60}-1}$

   $= \frac{250000\times0.005\times1.3489}{0.3489}$ = ₹ 4832.69

   ii. Given, P = ₹ 250000, i = $\frac{6}{12\times100}$ = 0.005 and n = 5 × 12 = 60

   Principal outstanding at beginning of 40th month

   $= \frac{\text{EMI}\left[(1+i)^{60-40+1}-1\right]}{i(1+i)^{60-40+1}} = \frac{4832.69\times\left[(1.005)^{21}-1\right]}{0.005\times(1.005)^{21}}$

   $= \frac{4832.69\times[1.1104-1]}{0.005\times1.1104} = \frac{4832.69\times0.1104}{0.005\times1.1104}$ = ₹ 96096.72

   iii. Given, P = ₹ 250000, i = $\frac{6}{12\times100}$ = 0.005 and n = 5 × 12 = 60

   Interest paid in 40th payment = $\frac{\text{EMI}\left[(1+i)^{60-40+1}-1\right]}{(1+i)^{60-40+1}}$

   = $\frac{4832.69\times\left[(1.005)^{21}-1\right]}{(1.005)^{21}} = \frac{4832.69\times0.1104}{1.1104}$ = ₹ 480.48

   **OR**

   Given, P = ₹ 250000, i = $\frac{6}{12\times100}$ = 0.005 and n = 5 × 12 = 60

   Principal paid in 40th payment = EMI - Interest paid in 40th payment

   = 4832.69 - 480.48 = ₹ 4352.21

38. i. 900
   ii. 2300
   iii. 300

   **OR**

   1300